Get the Logs to the Mill

Content Focus:

- Algebra

Possible Content Standards:

- CCSS.MATH.CONTENT.HSA.CED.A.3
- CCSS.MATH.CONTENT.HSF.LE.B.5

Mathematical Practices Focus:

- SMP1 - Make sense of problems and persevere in solving them.
- SMP2 - Reason abstractly and quantitatively.

Smarter-Balanced Assessment (SBA) Targets Addressed:

- Claim 2 Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.
- Claim 2 Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).

Overview

The purpose of this 3 ACT task is to provide students with an opportunity to solve a problem based on a real-world situation (SBA Claim 2 Target A and Target D). Due to the nature of the task, there are a variety of mathematical approaches students can take to successfully complete the task. Due to the nature of the task, there are a variety of mathematical approaches students can take to successfully complete the task; the mathematical approach presented in Act 3 of the task addresses CCSS.MATH.CONTENT.HSA.CED.A.3 (Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context), and CCSS.MATH.CONTENT.HSF.LE.B.5 (Interpret the parameters in a linear or exponential function in terms of a context). This performance task is intended for students with prior knowledge of creating equations with more than one variable. It would serve well as an assessment tool at the end of a unit.

The task is modeled after the task 3 ACT Fill 'Er Up by Graham Fletcher. In the task, students are presented with costs and distances for transporting raw timber to various mills for processing. Students then decide on necessary resources for finding the solution and are given time as a group to complete their work. The task concludes by having students examine the information provided in Act 3 to see if it answers their question.

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- Algebra

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Overview of task with specified standard addressed

The purpose of this 3 ACT task is to provide students with an opportunity to problem solve based on a real-world situation. The task is modeled after the 3 ACT Fill 'Er Up by Graham Fletcher. In the task, students are presented with a scenario faced by Washington State logging companies transporting timber to mills after harvest. Students then decide on necessary resources for finding the solution and are given time as a group to complete their work. The task concludes by having students examine the information provided in Act Three to see if it answers their question.

Learning Goal Statement

- Students will solve a range of complex well-posed problems in applied mathematics (SMP1).
- Students will solve a complex problem by making productive use of knowledge and problem-solving strategies (SMP2).

Success Criteria

- I can apply algebra concepts to solve a problem about timber harvest and transportation.
- I can build a function that models a relationship between two quantities.

Step By Step:

1. Materials

- Recording sheet, scratch paper, whiteboards, math journal (if applicable) for each group
- Technology to show videos/PowerPoint
- Student worksheet or notebooks and manipulatives

2. Pre-Planning

- Students will be identifying and solving their own student-generated problems based on the context provided by a hypothetical scenario, and a table of information about transporting logs from the logging site to a mill.
- Additional information is available to students as they come up with questions through the PowerPoint slides. You may choose to keep it in the PowerPoint format, print each question/answer on cards, or some other method. It is recommended that you not reveal the questions/answers unless students are asking. You can also choose to reveal the answers to specific groups or to the entire class.
- This lesson will include productive discussion that will open opportunities for multiple possible questions and math concepts. Students may need scaffolded supports or routines for productive discussions.
- Prepare access to materials such as scratch paper, math manipulatives, and other materials as you see fit for use as needed through the task.

3. Act 1: Introduction

- Read the learning goals aloud. Use the Think-Pair-Share strategy to have students respond to the prompt: "What connections or questions come to mind in relation to these learning goals?"
- Read PEI's Career Profile Card "Resource Forester."
- Ask students to Think-Pair-Share to answer the following questions: "What does a Resource Forester do?" "Why are trees harvested?" Elicit student ideas.

  Talk to students about the following: Trees are harvested for use in building materials, cellulose, and paper products. Trees must be milled after harvest, and mills can be miles from the harvest site. When timber is harvested, it can be complicated to maximize profits while considering all costs and limitations.
- Show the first "What do you notice and wonder?" slide.
- Ask students, "What kind of math questions can we ask about this information?"
- Show the second "What do you notice and wonder?" slide, with the Mill Location information chart.
- What do they notice about the information on this slide? What questions do they have? Remind students to use sentences like, "I notice, I see, I wonder, I want to ask…" Ask groups to record these initial thoughts on their "Notice and Wonder" chart.
- Ask students, "What kind of math questions can we answer using this information?"

- Allow time for students to come to a consensus about one question generated from the group ideas. Instruct students to analyze the questions generated to determine what information is needed in order to answer the generated questions.
- Have groups share questions with the class and discuss as a whole group ideas about what information is needed to answer the group's chosen question. Make connections to groups who have posed similar questions. Show the slide, "Timber companies have to maximize profits." Reinforce that these are questions the timber companies are looking to answer. Students are welcome to answer these questions but are encouraged to pursue their own generated questions as long as they stay connected to the facts provided.
- Prompt students to generate predictions for answers to the questions and post where visible to the class. These can be estimations; no calculation is required.

4. Act 2: Conflict

- Ask students, "Do you have everything you need to solve your problem?" Give students time to create a list of materials they will need or questions they have. Let students know that you have answers to common questions.
- Show how the Information Card provides the information from the scenario as well as examples of various algebraic strategies. Show students where to find this information.
- Show students the strategies available on the Information Card.
- Students can use the worksheet, math notebooks, graph paper, whiteboard, or manipulatives to organize and make sense of the data.
- Instruct student groups to record their thinking and math work on the recording sheet and other materials as needed. Inform students that this work will be collected as evidence of their learning. Consider using Flip or other technology to record student explanations.
- As students are working, be sure to ask questions about their thinking. Take note of different strategies students are using.
- Choose at least three (3) students to share their strategies with the class during Act 3. Make sure the strategies demonstrate math learning that align with learning goals.
- When student groups agree on an answer, instruct groups to answer the question ("Where should the logs be sold?") on their group recording sheets.
- Remind students of access to sentence frames on the Information Card for complete responses.
- Students can work with a partner to answer the questions, using their sentence stems.

5. Act 3: Resolution

- Allow at least three (3) students to share their groups' answers with a complete description of how they completed the task. Remind students of access to discussion frames for complete responses. Teacher moves: Pay attention to the solutions created by student groups. Select groups to present their solutions to the class and sequence the groups so they present from "least sophisticated" to "most sophisticated" solution methods.
- Ask questions that allow students to make connections between the different answer statements to the learning goal. For example: How were these approaches similar/different?

- Display the "Resolution" slides that provide answers to the questions from the initial prediction. If a group's question was the same, have students determine possible reasons for any differences between the answers. If a group's question was different, have students determine strategies for the presented solution that could have been used to support students in finding the answer to their problem.
- Ask students to rate their learning of the learning goals 0-10 (0 being you made no connection to the learning goals, 10 being you could teach this content) record what they learned.

Resolution:

This will take 100 loads regardless of which mill is chosen, since:

$$3{,}000\ ton\ /\ 30\ \frac{tons}{load} = 100\ loads$$

Aberdeen

Initial calculations show a profit of $290,000. This does not take into consideration that drivers want to work 12-hour days and will need to be compensated if they cannot work a full 12-hour day.

$$4\ hours \times 100\ loads = 400\ driving\ hours$$
$$Total\ cost\ for\ drivers: 400\ hours \times \$100/hour = \$40{,}000$$
$$Revenue\ from\ sale: \$110/ton \times 3{,}000\ tons = \$330{,}000$$
$$Calculated\ profit: \$330{,}000 - \$40{,}000 = \$290{,}000$$

After calculating the potential profit, you must consider the driver costs. Drivers can (and want to) work 12 hours per day. Because Aberdeen is a 4-hour round trip, each driver can do 3 loads per day.

$$3\ loads\ per\ driver \times 4\ drivers = 12\ loads\ per\ day$$
$$100\ total\ loads \div 12\ \frac{loads}{day} = 8.3\ days\ (9\ days)$$
$$400\ driving\ hours - \left(8\ days \times 12\ \frac{hours}{day} \times 4\ drivers\right) = 16\ hours$$

On the 9th day, two drivers will be working 8-hour days. You will need to use the bonus calculation to pay the drivers for this work.

$$(\$290{,}000 \times 0.0015)\ /\ 2\ drivers = \$217.50\ per\ driver$$

This bonus is equal to a little more than 2 hours of work, though the drivers are working 4 hours less than their desired 12-hour shift. It also must be subtracted from the calculated profit.

$$\boldsymbol{Final\ profit = \$290{,}000 - \$435 = \$289{,}565}$$

Amanda Park

Initial calculations show a profit of $285,000. This does not take into consideration that drivers want to work 12-hour days and will need to be compensated if they cannot work a full 12-hour day.

$$1.5\ hours \times 100\ loads = 150\ driving\ hours$$
$$Total\ cost\ for\ drivers: 150\ hours \times \$100/hour = \$15{,}000$$
$$Revenue\ from\ sale: \$100/ton \times 3{,}000\ tons = \$300{,}000$$
$$Calculated\ profit: \$300{,}000 - \$15{,}000 = \$285{,}000$$

After calculating the potential profit, you must consider the driver costs. Drivers can (and want to) work 12 hours per day. Because Amanda Park is a 1.5-hour round trip, each driver can do 8 loads per day.

$$8\ loads\ per\ driver\ \times\ 4\ drivers\ =\ 32\ loads\ per\ day$$

$$100\ total\ loads\ \div\ 32\ \frac{loads}{day}\ =\ 3.125\ days\ (4\ days)$$

$$150\ driving\ hours\ -\ (3\ days\ \times\ 12\ \frac{hours}{day}\ \times\ 4\ drivers)\ =\ 6\ hours$$

On the fourth day, one driver will be working a 6-hour day. You will need to use the bonus calculation to pay the driver for this work.

$$\$285{,}000\ \times\ 0.0015 = \$427.50$$

This bonus is equal to a little more than 4 hours of work, though the driver is working 6 hours less than their desired 12-hour shift. It also must be subtracted from the calculated profit.

$$Final\ profit\ =\ \$285{,}000\ -\ \$427.50\ =\ \$284{,}572.50$$

Bonus for this solution: Loggers are driving the least amount of miles and therefore causing the least amount of carbon emissions with this mill. If the students are considering other qualities besides which solution yields the most profit, then this option is the most "environmentally friendly."

Port Angeles

This is the least feasible of the three mill options. Initial calculations show a profit of $290,000. This does not take into consideration that drivers want to work 12-hour days and will need to be compensated if they cannot work a full 12-hour day.

$$7\ hours\ \times\ 100\ loads\ =\ 700\ driving\ hours$$

$$Total\ cost\ for\ drivers\text{: } 700\ hours\ \times\ \$100/hour\ =\ \$70{,}000$$

$$Revenue\ from\ sale\text{: } \$120/ton\ \times\ 3{,}000\ tons\ =\ \$360{,}000$$

$$Calculated\ profit\text{: } \$360{,}000\ -\ \$70{,}000\ =\ \$290{,}000$$

After calculating the potential profit, you must consider the driver costs. Drivers can (and want to) work 12 hours per day. Because Port Angeles is a 7-hour round trip, each driver can only do one load per day.

$$1\ load\ per\ driver\ \times\ 4\ drivers\ =\ 4\ loads\ per\ day$$

$$100\ total\ loads\ \div\ 4\ \frac{loads}{day}\ =\ 25\ days$$

One of the restrictions is time. Logs need to be delivered within 9 days of harvest. Not to mention, each driver will be working less than a 12-hour day and will probably need to be compensated for the time lost.

$$(\$290{,}000\ \times\ 0.0015)\ /\ 4\ drivers\ =\ \$108.75\ per\ driver$$

This bonus is equal to a little more than 1 hour of work, though the drivers are working 5 hours less than their desired 12-hour shift (a total of 125 hours over the 25-day job). This can make a job less desirable and harder to find drivers. It also must be subtracted from the calculated profit.

$$Final\ profit\ =\ \$290{,}000\ -\ \$435\ =\ \$289{,}565$$

Accessibility Strategies Used

- Scratch paper or white boards: Students can use blank paper to record thinking, complete calculation, create diagrams, etc.
- Manipulatives: Students can use any math manipulatives to support their problem solving.
- Small group collaborative work: Students work with peers to process their thinking, supporting each other. Intentional grouping may offer additional accessibility, though it is not necessary to complete the task.

Things to Consider

- The lesson can take different turns depending on the questions generated. Use this as an opportunity to reteach or extend different math concepts.
- There is opportunity for differentiation with intentional grouping of students by skill level, however this is not essential for students to meet the learning targets.
- The lesson can be split into two days where students create and find solutions to their questions on the first day and share their responses and discuss solutions on the second day.
- This task could be used as a math classroom tool in several ways:
    - as formative assessment pre- or post-instruction;
    - as an opportunity to practice new skills;
    - as practice for state tests; or
    - to help make connections to math in the world outside of the classroom.

Formative Assessment Process

- Clarify learning targets throughout the lesson. This is specifically done at the beginning and end, but is helpful at any point to further students' learning.
- Evidence of student learning is found in multiple areas of the lesson. The Group Recording Sheet and individual responses are concrete options. Teacher observations, student questions, and student discussion provide additional evidence of students meeting learning targets.
- Use observations of student thinking and other evidence as an opportunity for purposeful discussions around the math concepts. These can be opportunities to reteach or extend learning of math concepts.
- Feedback based on evidence of student learning should be provided to students throughout the lesson. This can happen as the teacher circulates the room, during class discussion, or on group or individual response sheets.

Strategies Used: In-depth look at teaching strategies used in the lesson

- 3 ACT Task
    - This is a whole-group task made up of three parts: Act 1 is an engaging situation that piques students' curiosity. Act 2 is where students seek information and work toward a solution, and Act 3 finishes the task by discussing solutions and tying the work back to the learning targets.
- Think-Pair-Share
    - With this strategy, students are given the opportunity to examine a prompt as an individual, then with a partner or small group, and finally sharing and listening to responses among the whole class.

- Notice/Wonder
  - This strategy allows students to unpack a problem or prompt before beginning to solve the problem or respond to the prompt. The purpose is to create a common experience and provide access for all students in an environment where students share their thoughts freely because there is no expectation to find the answer.
  - Find more about the Notice and Wonder strategy on the OER Commons: www.oercommons.org/courseware/lesson/79074/overview?section=1

Extensions and Connections learned from teacher implementation

- The challenge in the task is to apply what students know to a rich, in-context problem. While the mathematics they may use to solve the problem addresses earlier standards, the modeling of the problem brings up the complexity and difficulty for students.

Samples of Student Work

Coming soon.

Formative Assessment Rubric

Rubric Components	Point Scale			Student Score and Rationale
	3	2	1	
I can make a claim and justify it with mathematics for a solution to a problem about timber harvest and transportation.	Describe the claim clearly; show the mathematics to back up the solution.	Describe the claim, but description is minimal or incomplete or lacking support from the mathematics.	The claim is not clearly described; the mathematics are missing or do not support the claim.	
I can build a function that models a relationship between two quantities.	Wrote a function or rule that clearly describes the relationship between quantities.	Wrote a function that does not accurately or clearly describe the relationship between quantities (can explain reasoning).	Missing a function or rule that clearly describes the relationship between quantities.	
I can collaborate with others to model a timber harvest problem.		Respectful of others' ideas; actively include all members of the group; use talk moves and conversation strategies.	Disrespectful of others' ideas or actively exclude or discourage group members from participating.	

Presentation Materials - PowerPoint Slides

Get the Logs to the Mill

Management of Washington timber for processing.

3 ACT MATH TASK
Algebra

1

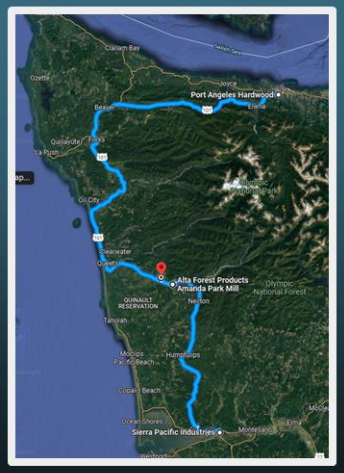
Learning Goals:

- ★ Students will solve a range of complex well-posed problems in applied mathematics (SMP1).
- ★ Students will solve a complex problem by making productive use of knowledge and problem-solving strategies (SMP2).

Success Criteria:

- ★ I can apply algebra concepts to solve a problem about timber harvest and transportation. (SMP1)
- ★ I can build a function that models a relationship between two quantities. (SMP2)

2



Scenario:

Logs have just been harvested from a field located in the Quinault Reservation at the red pin on the map.

There are three mills to choose from, located in Amanda Park, Port Angeles, and Aberdeen.

3

What do you notice and wonder?

We have 3000 tons of trees to get to the mill! We sell the logs to the mill, and pay log truck drivers to transport the logs. There are four trucks available to transport logs.

4

What do you notice and wonder?

We have 3000 tons of trees to get to the mill! We sell the logs to the mill, and pay log truck drivers to transport the logs. There are four trucks available to transport logs.

	Mill Location		
	Aberdeen	**Amanda Park**	**Port Angeles**
Round trip (including loading/unloading)	4 hours	1.5 hours	7 hours
What the mill will pay	$110/ton	$100/ton	$120/ton

5

Timber companies have to maximize profits.

★ Where should the freshly harvested timber be sold to be milled?

6

What other information do you need?

How long do we have to get the logs moved before they "expire"?	What can a log truck haul?	What is the hourly pay for each truck driver?	Is there a bonus for getting the job done in fewer days?
How much can a driver work in a day?	How much do drivers want/need to work in one day?	Can drivers get a bonus to encourage them to do a particular job?	*Click the question to collect additional information.*

7

Remember to talk about your ideas!

- I saw _____, so I connected that to _____ .
- We know _____ because _____ .
- Since _____ , then we can calculate _____ .

8

Discussion

Listen to each presenting group.

Consider what assumptions they have made. Do you agree or disagree with their methods?

Modify your own solution based on what you hear from presenting groups.

9

Resolution

It will take 100 loads to get these logs to the mill.

Aberdeen: It will take nine days to complete the job.

Calculated profit: $290,000
Cons: 2 drivers work 8 hour days on the last day. Would need to provide them a bonus to get them to take the job. Using the bonus calculation, the
final profit would be $289,565

10

Resolution

It will take 100 loads to get these logs to the mill.

Amanda Park: It will take four days to complete the job.

Calculated profit is $285,000
Cons: On the last day, you will need one driver to work a six hour day. You will need to give a bonus to this one driver. Using the bonus calculation, the
final profit would be $284,572.50.

11

Resolution

It will take 100 loads to get these logs to the mill.

Port Angeles: It will take 25 days to complete the job.

Calculated profit is $290,000
Cons: This is outside of the 9-day harvest-to-delivery constraint. Each day, drivers will only be able to make one delivery, and would need bonuses to make this attractive. You will need to give a bonus to this one driver. Using the bonus calculation, the
final profit would be $289,565.
Drivers would only earn a bonus of $108.75, barely the amount of one hour of work for 125 hours lost work.

12

Question:
Can drivers get a bonus to encourage them to do a particular job?

Answer:

Yes.

The calculation for the driver bonus is as follows:

Bonus money available is 0.15% of calculated profit before bonus.

(Example, if the profit was going to be $1000, then bonus available is $1.50. Therefore if you give the bonus, the actual profit would be $1000-$1.50=$998.50)

13

Question:
Is there a bonus for getting the job done in fewer days?

Answer:

No.

There is not a cash bonus.

14

Question:
How long do we have to get the logs moved before they "expire"?

Answer:

Logs need to get to the mill within 9 days for best quality.

15

Question:
How much do drivers want/need to work in one day?

Answer:

Drivers have options where to go to work.

They try to earn the most they can make per day by working 12 hours if possible.

16

Question:
What is the hourly pay for each truck driver?

Answer:

$100 per hour

17

Question:
How much can a driver work in a day?

Answer:

Drivers can work up to 12 hours a day.

They can't work more because of safety concerns.

18

Question:
What can a log truck haul?

Answer:

One truck can haul 30 tons at a time.

19

Algebra Math Performance Task: Get the Logs to the Mill

Recording Sheet

Name: _____ Group: _____ Date: _____

Notice	Wonder

1. Group Question:

2. Prediction:

3. Materials List:

4. Solution Thinking:

5. Final Answer:

6. Self-Reflection

Algebra Math Performance Task: Get the Logs to the Mill

Information Card

Scenario:

We have 3000 tons of trees to get to the mill! We sell the logs to the mill and pay log truck drivers to transport the logs. There are four trucks available to transport logs.

	Mill Location		
	Aberdeen	**Amanda Park**	**Port Angeles**
Round trip (including loading/unloading)	4 hours	1.5 hours	7 hours
What the mill will pay	$110/ton	$100/ton	$120/ton

Algebraic problem-solving strategies:

- ✓ **Make a list or table**
- ✓ **Draw a picture or construct a graph**
- ✓ **Guess and check**
- ✓ **Write an equation**
- ✓ **Act it out**
- ✓ **Use logical reasoning**
- ✓ **Solve part of the problem**
- ✓ **Work backwards**

Sentence stems for discussion:

I saw _____, so I connected that to _____ .

We know _____ because _____ .

Since _____ , then we can calculate _____ .

I think _____ because _____ .

I learned that _____ .

I agree because _____ .

I respectfully disagree because _____ .

Can you explain _____ ?

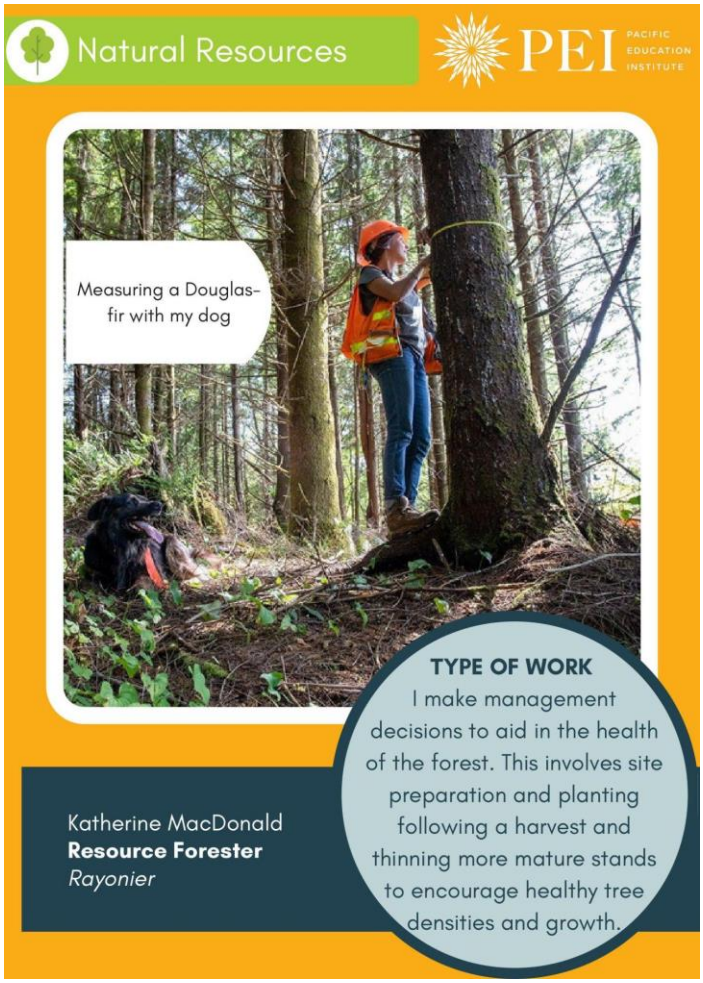
I can see connections between _____ and _____ because _____ .

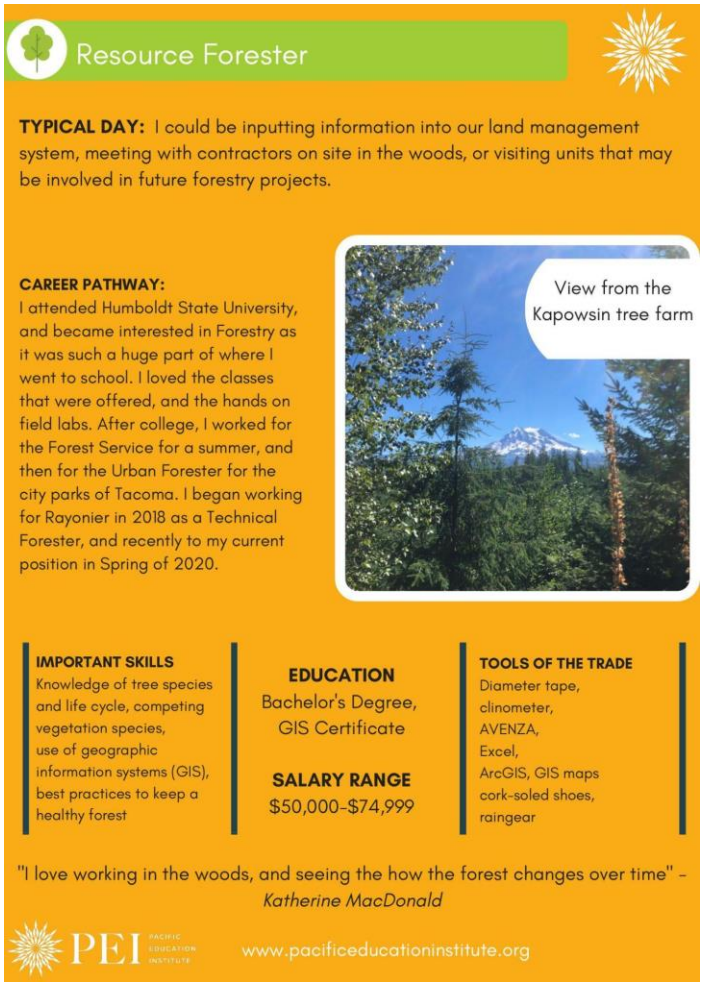
So, what I think _____ is saying is that _____ . Is that correct?

Additional Resources

Career Connections

Katherine MacDonald's career profile card can also be found on the PEI website at https://pacificeducationinstitute.org/wp-content/uploads/2020/10/CPC-NR-Katherine-MacDonald.pdf.





Community Resources Connection

Students may be unfamiliar with timber harvesting and transportation equipment. This collection of videos could help make the scenario more accessible:

- from YouTube user Log Safe Inc, Log Truck Loading Pacific North West in 2016: youtu.be/TBUmdMEGCpE?t=78
- from YouTube user northman logging, Unloading a Log Truck in 2019: youtu.be/636yCdelgUE?t=31
- from YouTube user Arunas195, The fastest wood loading unloading I had ever seen with VOLVO FH16 650 timbertruck: youtu.be/Yac6-DQlRWM