Replanting Timberland

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Content Focus:
- Algebra

Content Standard:
- CCSS.MATH.CONTENT.HSF.BF.A.1.A

Mathematical Practices:
- SMP1 - Make sense of problems and persevere in solving them.
- SMP2 - Reason abstractly and quantitatively.
- SMP4 - Model with mathematics.

Smarter-Balanced Assessment (SBA) Targets Addressed:
- Claim 2 Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.
- Claim 2 Target C: Interpret results in the context of a situation.
- Claim 2 Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).

Overview

The purpose of this 3 ACT task is to provide students with an opportunity to problem solve based on a real-world situation SBA Claim 2 Target A (Apply mathematics to solve problems arising in everyday life, society, and the workplace) and Target D (Identify important quantities in a practical situation and map their relationships). Due to the nature of the task, there are a variety of mathematical approaches students can take to successfully complete the task, however the mathematical approach presented in Act 3 of the task addresses CCSS.MATH.CONTENT.HSF.BF.A.1.A (Determine an explicit expression, a recursive process, or steps for calculation from a context). This performance task is intended for students with prior knowledge of creating equations with more than one variable. It would serve well as an assessment tool at the end of a unit.

The task is modeled after the 3 ACT Fill 'Er Up by Graham Fletcher. In the task, students are presented with the scenario in which a forester must plan for replanting a burned area for eventual timber harvest. Students then decide on necessary resources for finding the solution and are given time as a group to complete their work. The task concludes by having students examine the information provided in Act 3 to see if it answers their question.

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- SMP4 - Model with mathematics.

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- Claim 2 Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.
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- Claim 2 Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flow charts, or formulas).

Overview of task with specified standard addressed

The purpose of this 3 ACT task is to provide students with an opportunity to problem solve based on a real-world situation. The task is modeled after the 3 ACT Fill 'Er Up by Graham Fletcher. In the task, students are presented with a scenario faced by Washington State logging companies: replanting timber after a harvest or catastrophic burn. This is a complex and rich problem wherein students will need to determine an explicit expression or steps for calculation from the context provided (CCSS.MATH.CONTENT.HSF.BF.A.1.A). Students must decide on necessary resources for finding the solution and are given time as a group to complete their work. The task concludes by having students examine the information provided in Act 3 to see if it answers their question.

Learning Goal Statement

- Students will solve a range of complex well-posed problems in applied mathematics (SMP1).
- Students will solve a complex problem by making productive use of knowledge and problem-solving strategies (SMP2).
- Students will analyze complex, real-world scenarios (SMP4).

Success Criteria

- I can apply algebra concepts to solve a problem about replanting timberland. (SMP1, SMP2)
- I can build a function that models a relationship between two quantities. (SMP4)

Step By Step:

1. Materials

- Recording sheet, scratch paper, whiteboards, math journal (if applicable) for each group
- Technology to show videos/PowerPoint
- Student worksheet or notebooks and manipulatives

2. Pre-Planning

- Students will be identifying and solving their own student-generated problems based on the context provided by a hypothetical scenario, and information about timber replantation.
- Additional information is available to students as they come up with questions through the PowerPoint slides. You may choose to keep it in the PowerPoint format, print each question/answer on cards, or some other method. It is recommended that you not reveal the questions/answers unless students are asking. You can also choose to reveal the answers to specific groups as requested, or to the entire class.
- This lesson will include productive discussion that will open opportunities for multiple possible questions and math concepts. Students may need scaffolded supports or routines for productive discussions.
- Prepare access to materials such as scratch paper, math manipulatives, and other materials as you see fit for use as needed through the task.

3. Act 1: Introduction

- Read the learning goals aloud. Use the Think-Pair-Share strategy to have students respond to the prompt: “What connections or questions come to mind in relation to these learning goals?”
- Read PEI’s Career Profile Card “Silviculture Forester.”
- Ask students to Think-Pair-Share to answer the following questions: “What does a Silviculture Forester do?” “Why are trees replanted?” Elicit student ideas.

  Talk to students about the following: Trees are harvested for use in building materials, cellulose, and paper products. After harvest, trees must be replanted and replanting must consider ecosystems, botany, and economics, and it can be complicated to maximize profits while considering all of the costs and limitations.
- Show the first “Scenario” slide.
- Ask students, “What kind of math questions can we answer using this information?” Record student ideas in a public place.
- Show the second “Scenario” slide and revise the student-generated questions list.
- Allow time for students to come to a consensus about one question generated from the group ideas. You may want each group to explore their own question, or you could have a consensus question for the entire class. Instruct students to analyze the questions generated to determine what

information is needed in order to answer the generated questions. Students should record their question on the group recording sheet.

- Have groups share questions with the class and discuss as a whole group ideas about what information is needed to answer the group's chosen question. Make connections to groups who have posed similar questions. Show the slide, "Timber companies have to maximize profits." Reinforce that these are questions the timber companies are looking to answer. Students are welcome to answer these questions but are encouraged to pursue their own generated questions as long as they stay connected to the facts provided.
    - Timber companies must maximize profits. Some of the questions they may ask when replanting an area might be: How many seedlings of each species must be planted to ensure the desired number of trees at harvest? What should the spacing be for the plantings? Should we plant the 1-year-old or 2-year-old seedlings?
- Prompt students to generate predictions for answers to the questions and post where visible to the class. These can be estimations or qualitative; no calculation is required.

4. Act 2: Conflict

- Ask students, "Do you have everything you need to solve your problem?" Give students time to create a list of materials they will need or questions they have. Let students know that you have answers to common questions (in the PowerPoint slides).
- Show how the Information Card provides the information from the scenario as well as reminders of algebraic problem-solving strategies. Show students where to find this information.
- Students can use the worksheet, math notebooks, graph paper, whiteboard, or manipulatives to organize and make sense of the data.
- Instruct student groups to record their thinking and math work on the recording sheet and with other materials as needed. Inform students that this work will be collected as evidence of their learning. Consider using Flip or other technology to record student explanations.
- As students are working, be sure to ask questions about their thinking. Take note of different strategies students are using.
- Choose at least three (3) students or groups to share their strategies with the class during Act 3. Make sure the strategies demonstrate math learning that align with learning goals.
- When student groups agree on an answer, instruct groups to answer their question on their group recording sheets.
- Remind students of access to sentence frames on the Information Card for complete responses.
- Students can work with a partner to answer the questions, using their sentence stems.

5. Act 3: Resolution

- Allow at least three (3) students to share their groups' answers with a complete description of how they completed the task. Remind students of access to discussion frames for complete responses. Teacher moves: Pay attention to the solutions created by student groups. Select groups to present their solutions to the class and sequence the groups so they present from "least sophisticated" to "most sophisticated" solution methods.

- Ask questions that allow students to make connections between the different answer statements to the learning goal. For example: How were these approaches similar/different?
- Display the "Resolution" slides that provide answers to the questions from the initial prediction. If a group's question was the same, have students determine possible reasons for any differences between the answers. If a group's question was different, have students determine strategies for the presented solution could have been used to support students in finding the answer to their problem.
- Ask students to rate their learning of the learning goals 0-10 (0 being you made no connection to the learning goals, 10 being you could teach this content) record what they learned.

Algebra Math Performance Task: Replanting Timberland

Resolution:

Question: How many seedlings do I need?
Answer:

	Douglas Fir (DF)	Western Hemlock (WH)
Desired trees per acre:	$DF\ 60\% = 300\ \frac{trees}{acre} \times 0.60 = 180 \frac{DF\ trees}{acre}$	$WH\ 25\% = 300\ \frac{trees}{acre} \times 0.25 = 75\ \frac{WH\ trees}{acre}$
Overplanting value:	$\frac{180\ DF\ TPA}{0.92\ survival} = 196 \frac{DF\ trees}{acre}$	$\frac{75\ WH\ TPA}{0.81\ survival} = 93 \frac{WH\ trees}{acre}$
Number of seedlings:	$288\ acres \times 196 \frac{DF\ trees}{acre} = 56{,}488\ seedlings$	$288\ acres \times 93 \frac{WH\ trees}{acre} = 26{,}784\ seedlings$

	Western Red Cedar (WRC)
Desired trees per acre:	$WRC\ 15\% = 300\ \frac{trees}{acre} \times 0.15 = 45 \frac{WRC\ trees}{acre}$
Overplanting value:	$\frac{45\ DF\ TPA}{0.67\ survival} = 67 \frac{WRC\ trees}{acre}$
Number of seedlings:	$288\ acres \times 67 \frac{WRC\ trees}{acre} = 19{,}296\ seedlings$

$$\begin{array}{rr} & 56{,}488 \quad DF\ seedlings \\ & 26{,}784 \quad WH\ seedlings \\ + & 19{,}296 \quad WRC\ seedlings \\ \hline & \mathbf{102{,}528} \quad \boldsymbol{total\ seedlings} \end{array}$$

- - - - -

Question: How far apart should the trees be planted to ensure even distribution?
Answer:

$$\frac{102{,}528\ seedlings}{288\ acres} = 356 \frac{trees}{acre} \qquad \frac{43{,}560 ft^2}{356\ trees} = 122.34 \frac{ft^2}{tree}$$

$$\sqrt{122.34 ft^2} = \boldsymbol{11.06 ft\ between\ trees}$$

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Algebra Math Performance Task: Replanting Timberland

Question: What age seedlings should I order?

Answer:

This answer is constrained by how quickly the seedlings can be planted.

Option 1 - Order all seedlings of the same age

1-year-old seedlings:

$$1{,}000\ \frac{seedlings}{day} \times 5\ people = 5{,}000 \frac{seedlings}{day}$$

$$\frac{102{,}528\ total\ seedlings}{5{,}000\ seedlings\ per\ day} = 20.5\ days\ to\ plant$$

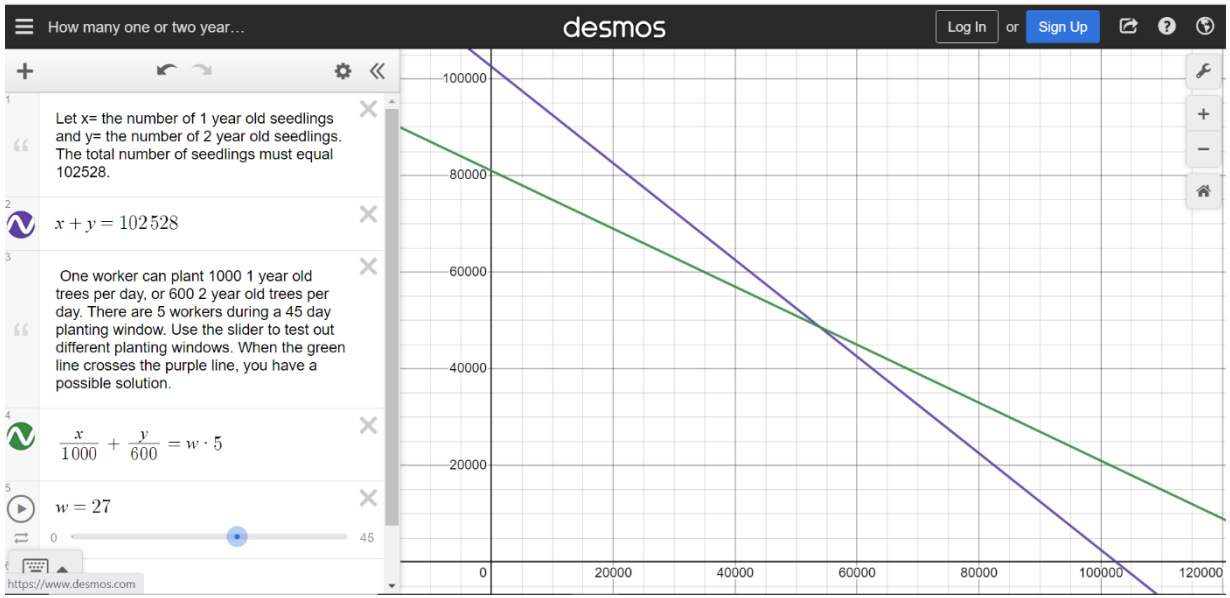
2-year-old seedlings:

$$600\ \frac{seedlings}{day} \times 5\ people = 3{,}000 \frac{seedlings}{day}$$

$$\frac{102{,}528\ total\ seedlings}{3{,}000\ seedlings\ per\ day} = 34.2\ days\ to\ plant$$

Option 2 - Order a combination of 1-year-old and 2-year-old seedlings

Please see the interactive graphical explanation available here: https://www.desmos.com/calculator/cgn8qgie5l



Let *x* be the number of 1-year-old seedlings, and *y* be the number of 2-year-old seedlings. The total number of seedlings must equal 102,528.

$$x + y = 102{,}528\ (\rightsquigarrow)$$

One worker can plant 1,000 one-year-old seedlings per day, or 600 two-year-old seedlings per day. There are 5 workers, and the planting season is at maximum 45 days. Therefore, the work cannot exceed 225 person days. However, it is possible the planting window could be shortened by late snowmelt or by early summer heat. So, it is a good idea to consider a solution where the planting window might be shorter.

$$\frac{x}{1{,}000} + \frac{y}{600} = 5 \cdot w,$$

where *w* represents days in the planting window ($\rightsquigarrow$)

When both equations are plotted on the same coordinate plane, the best solution will be found where the two lines intersect. If *w*, the planting window, is between 21 and 34, then the intersection of the two lines is the best solution.

Algebra Math Performance Task: Replanting Timberland

Accessibility Strategies Used

- Scratch paper or white boards: Students can use blank paper to record thinking, complete calculation, create diagrams, etc.
- Manipulatives: Students can use any math manipulatives to support their problem solving.
- Small group collaborative work: Students work with peers to process their thinking, supporting each other. Intentional grouping may offer additional accessibility, though it is not necessary to complete the task.

Things to Consider

- The lesson can take different turns depending on the questions generated. Use this as an opportunity to reteach or extend different math concepts.
- There is opportunity for differentiation with intentional grouping of students by skill level, however this is not essential for students to meet the learning targets.
- The lesson can be split into two days where students create and find solutions to their questions on the first day and share their responses and discuss solutions on the second day.
- This task could be used as a math classroom tool in several ways:
    - as formative assessment pre- or post-instruction;
    - as an opportunity to practice new skills;
    - as practice for state tests; or
    - to help make connections to math in the world outside of the classroom.

Formative Assessment Process

- Clarify learning targets throughout the lesson. This is specifically done at the beginning and end, but is helpful at any point to further students' learning.
- Evidence of student learning is found in multiple areas of the lesson. The Group Recording Sheet and individual responses are concrete options. Teacher observations, student questions, and student discussion provide additional evidence of students meeting learning targets.
- Use observations of student thinking and other evidence as an opportunity for purposeful discussions around the math concepts. These can be opportunities to reteach or extend learning of math concepts.
- Feedback based on evidence of student learning should be provided to students throughout the lesson. This can happen as the teacher circulates the room, during class discussion, or on group or individual response sheets.

Strategies Used: In-depth look at teaching strategies used in the lesson

- 3 ACT Task
    - This is a whole-group task made up of three parts: Act 1 is an engaging situation that piques students' curiosity. Act 2 is where students seek information and work toward a solution, and Act 3 finishes the tasks by discussing solutions and tying the work back to the learning targets.
- Think-Pair-Share

- With this strategy, students are given the opportunity to examine a prompt as an individual, then with a partner or small group, and finally sharing and listening to responses among the whole class.

- Notice/Wonder
  - This strategy allows students to unpack a problem or prompt before beginning to solve the problem or respond to the prompt. The purpose is to create a common experience and provide access for all students in an environment where students share their thoughts freely because there is no expectation to find the answer.
  - Find more about the Notice and Wonder strategy on the OER Commons: www.oercommons.org/courseware/lesson/79074/overview?section=1

Extensions and Connections learned from teacher implementation

- The challenge in the task is to apply what students know to a rich, in-context problem. While the mathematics they may use to solve the problem addresses earlier standards, the modeling of the problem brings up the complexity and difficulty for students.
- **Lesson Extension Option:** Survival Survey
  - Act 1: At two years old, it is necessary to do a survival survey to see how many trees that were planted survived. A distributed sample of one plot/acre is desired across the entire landscape. How long will it take a person to complete the job?
  - Act 2: To do this I know that one person can do 35 plots per day on gentle ground, 28 plots per day on marginally steep ground and only 20 plots per day on steep rugged terrain. The 288 acres is comprised of a mix of: 28% gentle ground, 46% marginally steep ground and 26% steep rugged terrain.
  - Act 3: Resolution

$$288\ acres \times 0.28 = 81\ plots\ on\ gentle\ ground$$
$$\frac{81\ plots}{35\ plots\ per\ day} = 2.3\ days$$

$$288\ acres \times 0.46 = 132\ plots\ on\ marginal\ ground$$
$$\frac{132\ plots}{28\ plots\ per\ day} = 4.7\ days$$

$$288\ acres \times 0.26 = 75\ plots\ on\ rugged\ steep\ terrain$$
$$\frac{75\ plots}{20\ plots\ per\ day} = 3.8\ days$$

$$Total = 10.8\ days$$

- **Lesson Extension Option:** Diameter of the Trees

  Standard for this is CCSS.MATH.CONTENT.7.G.B.4, Know the formulas for the area and circumference of a circle and use them to solve problems.
  - Act 1: Once trees are around 25 years old, the diameter of the trees is measured. What is the diameter of the trees one foot above the ground when they are 25 years old? (Douglas-fir, Western Hemlock, and Western Red Cedar)
  - Act 2: The average circumference of the trees at age 25 at 1' above the ground level is:
    - Douglas-fir: 30"
    - Western Hemlock: 36"

- Western Red Cedar: 35”
- Diameter is circumference divided by π.

- Act 3: Resolution

$$Douglas\ fir: \frac{30\ in}{\pi} = 9.55\ in$$

$$Western\ hemlock: \frac{36\ in}{\pi} = 11.46\ in$$

$$Western\ red\ cedar: \frac{35\ in}{\pi} = 11.14\ in$$

- **Lesson Extension Option:** Annual Growth Rates

Standard for this is CCSS.MATH.CONTENT.8.F.B.4, Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- Act 1: The trees were measured 2 years after planting for a survival survey. Then, 10 years after planting, the trees were measured again. What was the average height growth per year between the survival survey and the 10-year remeasurement? What is the predicted height for each tree 25 years after planting?
- Act 2: When measuring the survival plots, tree heights of the seedlings was also measured and the average tree height by species after the second growing season in the ground was:
  - Douglas-fir, 4.8 ft; Western Hemlock, 5.2 ft; Western Red Cedar, 3.9 ft

  After the tenth growing season, the stand was remeasured with the following average heights:
  - Douglas-fir, 33.1 ft; Western Hemlock, 29.6 ft; Western Red Cedar, 24.8 ft
- Act 3: Resolution

  Calculate the average growth per year by dividing change in height by elapsed time in years.

$$Douglas\ fir: \frac{33.1ft - 4.8ft}{10\ years - 2\ years} = 3.5ft\ average\ growth\ per\ year$$

$$Western\ hemlock: \frac{29.6ft - 5.2ft}{10\ years - 2\ years} = 3.05ft\ average\ growth\ per\ year$$

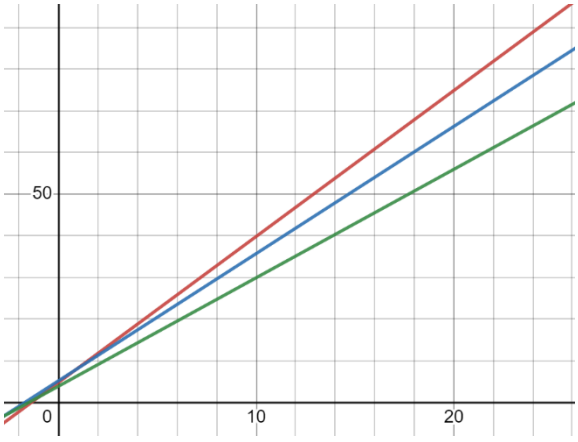
$$Western\ red\ cedar: \frac{24.8ft - 3.9ft}{10\ years - 2\ years} = 2.6ft\ average\ growth\ per\ year$$

Use the 2-year height as *y*-intercept to graph the equations.

$$Douglas\ fir: d(x) = 4.8 + 3.5x$$

$$Western\ hemlock: h(x) = 5.2 + 3.05x$$

$$Western\ red\ cedar: r(x) = 3.9 + 2.6x$$



Predict the heigh at 25 years.

Recall that the *y*-intercept is when the tree is 2-years old. The height at 25 years would be read at $x = 23\ years.$

$$Douglas\ fir: d(23) = 85.3ft$$

$$Western\ hemlock: h(23) = 75.4ft$$

$$Western\ red\ cedar: r(23) = 63.7ft$$

- **Lesson Extension Option:** Thinning the Trees

  Standard for this is CCSS.MATH.CONTENT.HSF.LE.B.5, Interpret the parameters in a linear or exponential function in terms of a context.

  - Act 1: At age 25 it is desired to remove 50% of the trees to thin the growing timber stand in order to have a more healthy forest and get better growth on the remaining trees. How many trees are remaining by species at age 25? How many trees will remain of each species after removing 50% of the trees?
  - Act 2: The normal mortality rate across all species is 2% per decade. Assume after 2 years we have the following mix per each acre (per the original Performance Task):
    - $60\%\ Douglas\ fir\ (DF) \times 300 = \ 180\ DF\ trees$
    - $25\%\ Western\ hemlock\ (WH) \times 300 = \ 75\ WH\ trees$
    - $15\%\ Western\ Red\ Cedar\ (WRC) \times 300 = \ 45\ WRC\ trees$
  - Act 3: Resolution

    These populations will decay by 2% per decade for 2.3 decades. 25 years after planting we expect these populations per acre:

    $$180\ DF\ trees\ (0.98)^{2.3} = 171.8\ or\ 172\ Douglas\ fir\ trees$$

    $$75\ WH\ trees\ (0.98)^{2.3} = 71.6\ or\ 72\ Western\ hemlock\ trees$$

    $$45\ WRC\ trees\ (0.98)^{2.3} = 42.9\ or\ 43\ Western\ red\ cedar\ trees$$

    Based on these calculations, 25 years after planting we will thin the trees by 50%.

    $$Douglas\ fir: 0.50 \times 172 = 86\ trees\ thinned, 86\ trees\ kept$$

    $$Western\ hemlock: 0.50 \times 72 = 36\ trees\ thinned, 36\ trees\ kept$$

    $$Western\ red\ cedar: 0.50 \times 43 = 21\ trees\ thinned, 22\ trees\ kept$$

- **Lesson Extension Option:** Selling Thinned Trees

  Standard for this is CCSS.MATH.CONTENT.HSN.Q.A.1, use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

  - Act 1: All the logs generated in the thinning will be delivered to a pulp mill to make paper from the logs that are shipped. What is the round-trip time to the mill? What is the total cost to haul 1 load of logs? If the average log truck can haul 29 tons per load, then what is the cost per ton of the haul?
  - Act 2: All the logs generated in the thinning will be delivered to a pulp mill to make paper from the logs that are shipped. The cost of harvesting the logs is $20/ton to cut, move and load the logs on a truck. Trucking costs $98/hour. It also takes 30 minutes to load logs on a truck and 20 minutes to unload the logs at the pulp mill. The timber is located 53 miles from the mill on the following road system:
    - Timber Area on rocked road to main line haul road: 8 miles
    - Main line haul road to State Highway: 14 miles
    - State highway to Pulp Mill: 31 miles
    - The average speed trucks can go on each road is:
      - Rocked road: 20 mph
      - Main line: 35 mph
      - State Highway: 55 mph

- Act 3: Resolution

  What is the round-trip time to the mill? What is the total cost to haul 1 load of logs? Since we are dealing with round trip trucking the miles for each segment of road must be doubled, so we would be calculating for 16 miles of rocked road, 28 miles of mainline and 62 miles of highway. In addition to driver cost, we would also need to include cost to harvest the logs.

  $$\frac{16\ miles}{20\ mph} + \frac{28\ miles}{35\ mph} + \frac{62\ miles}{55\ mph} + \frac{50\ minutes\ loading\ and\ unloading}{60\ minutes}$$

  $$= 0.8\ hours + 0.8\ hours + 1.27\ hours + 0.83\ hours = \mathbf{3.7\ hours}$$

  $$Cost\ of\ haul = 3.7\ hours \times \$98\ per\ hour + \$20\ per\ ton \times 29\ tons\ per\ load$$

  $$= \$362.60\ for\ driver\ pay + \$580\ harvest\ cost = \mathbf{\$942.60}$$

  If the average log truck can haul 29 tons/load, what is the cost per ton of the haul?

  $$\frac{\$362.60\ driver\ pay}{29\ tons} + \$20\ harvest\ cost\ per\ ton = \mathbf{\$32.50\ per\ ton}$$

Samples of Student Work

Coming soon.

Formative Assessment Rubric

Rubric Components	Point Scale			Student Score and Rationale
	3	2	1	
I can make a claim and justify it with mathematics for a solution to a problem about timber replantation.	Describe the claim clearly; show the mathematics to back up the solution.	Describe the claim, but description is minimal or incomplete or lacking support from the mathematics.	The claim is not clearly described; the mathematics are missing or do not support the claim.	
I can build a function that models a relationship between two quantities.	Wrote a function or rule that clearly describes the relationship between quantities.	Wrote a function that does not accurately or clearly describe the relationship between quantities (can explain reasoning).	Missing a function or rule that clearly describes the relationship between quantities.	
I can collaborate with others to model a timber replantation problem.		Respectful of others' ideas; actively include all members of the group; use talk moves and conversation strategies.	Disrespectful of others' ideas or actively exclude or discourage group members from participating.	

Presentation Materials - PowerPoint Slides

Replanting Timber

Management of Washington forests for harvest.

3 ACT MATH TASK
Algebra

1

Learning Goals:

- ★ Students will solve a range of complex well-posed problems in applied mathematics (SMP1).
- ★ Students will solve a complex problem by making productive use of knowledge and problem-solving strategies (SMP2).
- ★ Students will analyze complex, real-world scenarios (SMP4).

Success Criteria:

- ★ I can apply algebra concepts to solve a problem about timber replantation. (SMP1, SMP2)
- ★ I can build a function that models a relationship between two quantities. (SMP4)

2

Scenario:

Bill needs to order seedlings for planting trees to restore 288 acres of burned timberland. Not every seedling that is planted will survive. He can order 1-year-old seedlings and 2-year-old seedlings. Bill has a work crew of 5 people available for the spring planting season to get the seedlings in the ground before the heat of summer.

3

We would like to have survival of 300 trees/acre (TPA) at the end of 2 years of the following mix:

60% Douglas-fir (DF)
25% Western Hemlock (WH)
15% Western Red Cedar (WRC)

Scenario

What does the forester want the forest to look like after two years?

4

Timber companies have to maximize profits.

There is a cost associated with the planting of new seedlings.
As you work through your solution, consider:

- ★ How many seedlings do I need?
- ★ What are the costs?
- ★ Where can losses occur?
- ★ How many seedlings are needed?
- ★ What should the spacing be?
- ★ What age of seedlings should be planted?

5

Question:

What is your replanting plan?

Be prepared to explain your reasoning.

6

What other information do you need?

How large is an acre?	What is the planting season?
How many trees can be planted daily?	What are the tree species survival rates?

Click the question to collect additional information.

7

Remember to talk about your ideas!

- I saw _____, so I connected that to _____ .
- We know _____ because _____ .
- Since _____ , then we can calculate _____ .

8

Discussion

Listen to each presenting group.

Consider what assumptions they have made. Do you agree or disagree with their methods?

Modify your own solution based on what you hear from presenting groups.

9

Resolution

How many seedlings do I need?

At the end of 2 years, you need 300 trees per acre (TPA).
The trees need to be in the following ratio
60% DF , 25% WH , 15% WRC
Each species has a unique survival rate that is less than 100%, so you will need to plant a surplus of seedlings to ensure 300 trees per acre outcome.

1. Figure out how many of each species you want to have after two years.
2. Figure out how many seedlings to plant to ensure that many trees will survive.
3. Calculate how many seedlings are needed to plant the 288 acre site.

10

Resolution

How many seedlings do I need?

Douglas Fir (DF)

$$DF\ 60\% = (300 TPA \times 0.60) = 180\ DF\ TPA$$

$$DF\ survival\ rate: 92\%$$

$$Need\ to\ plant\ per\ acre = \frac{180}{0.92} = 196\ DF\ TPA$$

$$288\ acres \times 196\ DF\ TPA = 56,448\ DF\ seedlings$$

11

Resolution

How many seedlings do I need?

Western Hemlock (WH)

$$WH\ 25\% = (300\ TPA \times 0.25) = 75\ WH\ TPA$$

$$WH\ survival\ rate: 81\%$$

$$Need\ to\ plant\ per\ acre = \frac{75}{0.81} = 93\ WH\ TPA$$

$$288\ acres \times 93\ WH\ TPA = 26,784\ WH\ seedlings$$

12

Resolution

How many seedlings do I need?

Western Red Cedar (WRC)

$$WRC\ 15\% = (300\ TPA \times 0.15) = 45\ WRC\ TPA$$

$$WRC\ survival\ rate: 67\%$$

$$Need\ to\ plant\ per\ acre = \frac{45}{0.67} = 67\ WRC\ TPA$$

$$288\ acres \times 67\ WRC\ TPA = 19,296\ WRC\ seedlings$$

13

Resolution

How many seedlings do I need?

Total seedlings needed for replanting:

$$\begin{array}{rrl} & 56,448 & DF\ seedlings \\ & 26,784 & WH\ seedlings \\ + & 19,296 & WRC\ seedlings \\ \hline & 102,528 & total\ seedlings \end{array}$$

14

Resolution

How far apart should the trees be planted to ensure even distribution?

11.06 ft spacing between plantings if on a grid

1. Calculate the number of trees per acre
2. Calculate the number of square feet per tree
3. Find the distance between trees by calculating the square root.

$$\frac{102,528\ seedlings}{288\ acres} = 356\ \frac{trees}{acre} \qquad \frac{43,560\ ft^2}{356\ trees} = 122.34\ \frac{ft^2}{tree}$$

$$\sqrt{122.34\ ft^2} = 11.06\ ft$$

15

Resolution

What age seedlings should I order?

Since there are up to 45 days in the season, both of these solutions are feasible!

Option 1: Order all seedlings of the same age

1 year-old seedlings:

$$1,000\ seedlings\ per\ day \times 5\ people = 5,000\ seedlings\ per\ day$$

$$\frac{102,528\ total\ seedlings}{5,000\ seedlings\ per\ day} = 20.5\ days$$

2 year-old seedlings:

$$600\ seedlings\ per\ day \times 5\ people = 3,000\ seedlings\ per\ day$$

$$\frac{102,528\ total\ seedlings}{3,000\ seedlings\ per\ day} = 34.2\ days$$

16

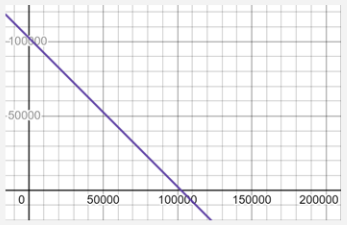
Resolution

What age seedlings should I order?

Option 2: Order a combination of 1-year-old and 2 year-old seedlings

Let x = the number of 1-year-old seedlings and y = the number of 2-year-old seedlings. The total number of seedlings must equal 102,528.

$$x + y = 102,528$$



17

Resolution

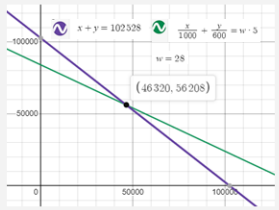
What age seedlings should I order if I'm worried the planting season isn't going to be 45 planting days? (link to interactive graph)

Example solution: When the planting window is set for 28 days, then it is best to have 46,320 1 year olds and 56,208 2 year olds.

Option 2: Order a combination of 1-year-old and 2-year-old seedlings

One worker can plant 1,000 1-year-old trees per day, or 600 2-year-old trees per day. There are 5 workers during a planting window, w, that could be from 0 to 45 days. Make w less than 45 to shorten the planting window.

$$\frac{x}{1000} + \frac{y}{600} = w \cdot 5$$



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Question:

How large is an acre?

Answer:

43,560 square feet (ft^2)

19

Question:

What is the planting window?

Answer:

For the particular 288 acres in question, the area is usually covered with snow until May 15. After July 15, the soil is too dry to support a newly planted seedling. There are about 9 weeks, so that is 45 maximum work days per person during the planting season. ***It is possible that the snow might not melt by the May 15th, and that hot weather comes before July 15.***

May						
Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

June						
Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

July						
Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

20

Question:
How many trees can be planted each day?

Answer:

One tree planter can plant 1,000 one-year-old seedlings per day.

One tree planter can plant 600 two-year-old seedlings per day.

Planters work 5 days per week.

21

Question:
What are the tree species survival rates?

Answer:

	Tree Species		
	Douglas Fir (DF)	Western Hemlock (WH)	Western Red Cedar (WRC)
seedling survival rate	92%	81%	67%

** Because the area planted is at high elevation, planting 2-year-old trees instead of the normal 1-year-old trees we can ensure better growth if they survive.*

22

Extension: Survival Survey Act 1

At two years old, it is necessary to do a survival survey to see how many trees that were planted survived. A distributed sample of one plot/acre is desired across the entire landscape. How long will it take a person to complete the job?

23

Extension: Survival Survey Act 2

To do this I know that one person can do 35 plots per day on gentle ground, 28 plots per day on marginally steep ground and only 20 plots per day on steep rugged terrain.

The 288 acres is comprised of a mix of: 28% gentle ground, 46% marginally steep ground and 26% steep rugged terrain.

24

Extension: Survival Survey Act 3

$$288\ acres \times 0.28 = 81\ plots\ on\ gentle\ ground$$
$$\frac{81\ plots}{35\ plots\ per\ day} = 2.3\ days$$

$$288\ acres \times 0.46 = 132\ plots\ on\ marginal\ ground$$
$$\frac{132\ plots}{28\ plots\ per\ day} = 4.7\ days$$

$$288\ acres \times 0.26 = 75\ plots\ on\ rugged\ steep\ terrain$$
$$\frac{75\ plots}{20\ plots\ per\ day} = 3.8\ days$$

$$Total = 10.8\ days$$

25

Extension: Diameter of the trees Act 1

Once trees are around 25 years old, the diameter of the trees is measured. What is the diameter of the trees one foot above the ground when they are 25 years old?

Douglas-fir, Western Hemlock, and Western Red Cedar

26

Extension: Diameter of the trees ACT 2

The average circumference of the trees at age 25 at 1' above the ground level is:

Douglas-fir: 30"

Western HemlocK: 36"

Western Red Cedar: 35"

Diameter is circumference divided by π.

27

Extension: Diameter of the trees Act 3

The average diameter of the trees at age 25 at 1' above the ground level is:

$$Douglas\ fir: \frac{30\ in}{\pi} = 9.55\ in$$

$$Western\ hemlock: \frac{36\ in}{\pi} = 11.46\ in$$

$$Western\ red\ cedar: \frac{35\ in}{\pi} = 11.14\ in$$

28

Extension: Annual Growth rates Act 1

The trees were measured 2 years after planting for a survival survey. Then, 10 years after planting, the trees were measured again.

What was the average height growth per year between the survival survey and the 10-year remeasurement?

What is the predicted height for each tree 25 years after planting?

29

Extension: Annual Growth rates Act 2

When measuring the survival plots, tree heights of the seedlings was also measured and the average tree height by species after the second growing season in the ground was:

Douglas-fir: 4.8'
Western Hemlock: 5.2'
Western Red Cedar: 3.9'

After the tenth growing season the stand was remeasured with the following heights:

Douglas-fir: 33.1'

Western Hemlock: 29.6'

Western Red Cedar: 24.8'

30

Extension: Annual Growth rates Act 3

ANSWER for average rate of change:

Calculate (Year 10 height - Year 2 height) divided by the elapsed time in years.

$$Douglas\ fir: \frac{33.1ft - 4.8ft}{10\ years - 2\ years} = 3.5ft\ average\ growth\ per\ year$$

$$Western\ hemlock: \frac{29.6ft - 5.2ft}{10\ years - 2\ years} = 3.05ft\ average\ growth\ per\ year$$

$$Western\ red\ cedar: \frac{24.8ft - 3.9ft}{10\ years - 2\ years} = 2.6ft\ average\ growth\ per\ year$$

31

Extension: Annual Growth rates Act 3

Prediction of height at 25 years:

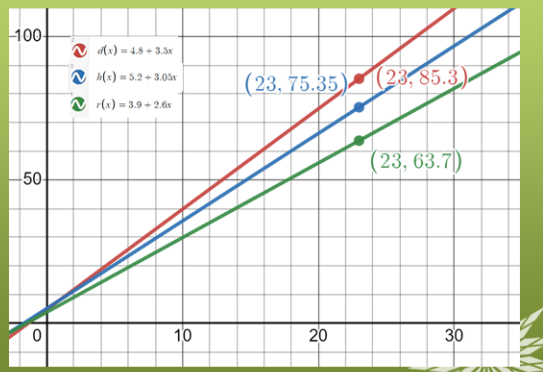
Use the 2 year height as your *y*-intercept, and find the height when *x* = 23.

(link to interactive graph)

Douglas fir = 85.3 ft tall

Western hemlock = 75.35 ft tall

Western red cedar = 63.7 ft tall



32

Extension: Thinning the trees Act 1

At age 25 it is desired to remove 50% of the trees to thin the growing timber stand in order to have a more healthy forest and get better growth on the remaining trees.

How many trees are remaining by species at age 25?

How many trees will remain of each species after removing 50% of the trees?

33

Extension: Thinning the trees Act 2

The normal mortality rate across all species is 2% per decade.

Assume after 2 years we have the following mix per each acre (from the very first problem):

- 60% Douglas-fir (DF)*300= 180 DF trees
- 25% Western Hemlock (WH)*300= 75 WH trees
- 15% Western Red Cedar (WRC)*300= 45 WRC trees

34

Extension: Thinning the trees Act 3

The normal mortality rate across all species is 2% per decade.

These populations will decay by 2% per decade for 2.3 decades. 25 years after planting we expect these populations per acre.

$$180\ DF\ trees\ (0.98)^{2.3} = 171.8\ or\ 172\ Douglas\ fir\ trees$$
$$75\ WH\ trees\ (0.98)^{2.3} = 71.6\ or\ 72\ Western\ hemlock\ trees$$
$$45\ WRC\ trees\ (0.98)^{2.3} = 42.9\ or\ 43\ Western\ red\ cedar\ trees$$

35

Extension: Thinning the trees ACT 3

The normal mortality rate across all species is 2% per decade.

25 years after planting we expect these populations per acre, so we will thin the trees by 50%.

$$Douglas\ fir: 0.50 \times 172 = 86\ trees\ thinned, 86\ trees\ kept$$
$$Western\ hemlock: 0.50 \times 72 = 36\ trees\ thinned, 36\ trees\ kept$$
$$Western\ red\ cedar: 0.50 \times 43 = 21\ trees\ thinned, 22\ trees\ kept$$

36

Extension: Selling Thinned Trees Act 1

All the logs generated in the thinning will be delivered to a pulp mill to make paper from the logs that are shipped. What is the round-trip time to the mill? What is the total cost to haul 1 load of logs? If the average log truck can haul 29 tons per load, then what is the cost per ton of the haul?

37

Extension: Selling Thinned Trees Act 2

All the logs generated in the thinning will be delivered to a pulp mill to make paper from the logs that are shipped. The cost of harvesting the logs is $20/ton to cut, move and load the logs on a truck. Trucking costs $98/hour. It also takes 30 minutes to load logs on a truck and 20 minutes to unload the logs at the pulp mill. The timber is located 53 miles from the mill on the following road system:

Timber Area on rocked road to main line haul road: 8 miles
Main line haul road to State Highway: 14 miles
State highway to Pulp Mill: 31 miles

The average speed trucks can go on each road is:

20 mph on rocked road ; 35 mph on main line ; 55 mph on state highway

38

Extension: Selling Thinned Trees Act 3

What is the round-trip time to the mill? What is the total cost to haul 1 load of logs?

Since we are dealing with round trip trucking the miles for each segment of road must be doubled. Therefore, 16 miles of rocked road, 28 miles of mainline and 62 miles of highway. In addition to driver cost, also include cost to harvest the logs.

$$\frac{16\ miles}{20\ mph}+\frac{28\ miles}{35\ mph}+\frac{62\ miles}{55\ mph}+\frac{50\ minutes\ loading\ and\ unloading}{60\ minutes}$$

$$= 0.8\ hours + 0.8\ hours + 1.27\ hours + 0.83\ hours = \mathbf{3.7\ hours}$$

$$Cost\ of\ haul = 3.7\ hours \times \$98\ per\ hour + \$20\ per\ ton \times 29\ tons\ per\ load$$

$$= \$362.60\ for\ driver\ pay + \$580\ harvest\ cost = \mathbf{\$942.60}$$

39

Extension: Selling thinned trees Act 3

If the average log truck can haul 29 tons/load, what is the cost per ton of the haul?

$$\frac{\$362.60\ driver\ pay}{29\ tons} + \$20\ harvest\ cost\ per\ ton = \mathbf{\$32.50\ per\ ton}$$

40

Algebra Math Performance Task: Replanting Timberland

Recording Sheet

Name: _____ Group: _____ Date: _____

Notice	Wonder

1. Group Question:

2. Prediction:

3. Materials List:

4. Solution Thinking:

5. Final Answer:

6. Self-Reflection

Algebra Math Performance Task: Replanting Timberland

Information Card

Scenario:

Bill needs to order seedlings for planting trees to restore 288 acres of burned timberland. Not every seedling that is planted will survive. He can order either 1-year-old seedlings or 2-year-old seedlings. Bill has a work crew of 5 people available for the spring planting season to get the seedlings in the ground before the heat of summer.

Desired species diversity, at 300 trees/acre		
Tree Species		
Douglas Fir (DF)	**Western Hemlock (WH)**	**Western Red Cedar (WRC)**
60%	25%	15%

Algebraic problem-solving strategies:

- ✓ **Make a list or table**
- ✓ **Draw a picture or construct a graph**
- ✓ **Guess and check**
- ✓ **Write an equation**
- ✓ **Act it out**
- ✓ **Use logical reasoning**
- ✓ **Solve part of the problem**
- ✓ **Work backwards**

Sentence stems for discussion:

I saw _____, so I connected that to _____ .

We know _____ because _____ .

Since _____ , then we can calculate _____ .

I think _____ because _____ .

I learned that _____ .

I agree because _____ .

I respectfully disagree because _____ .

Can you explain _____ ?

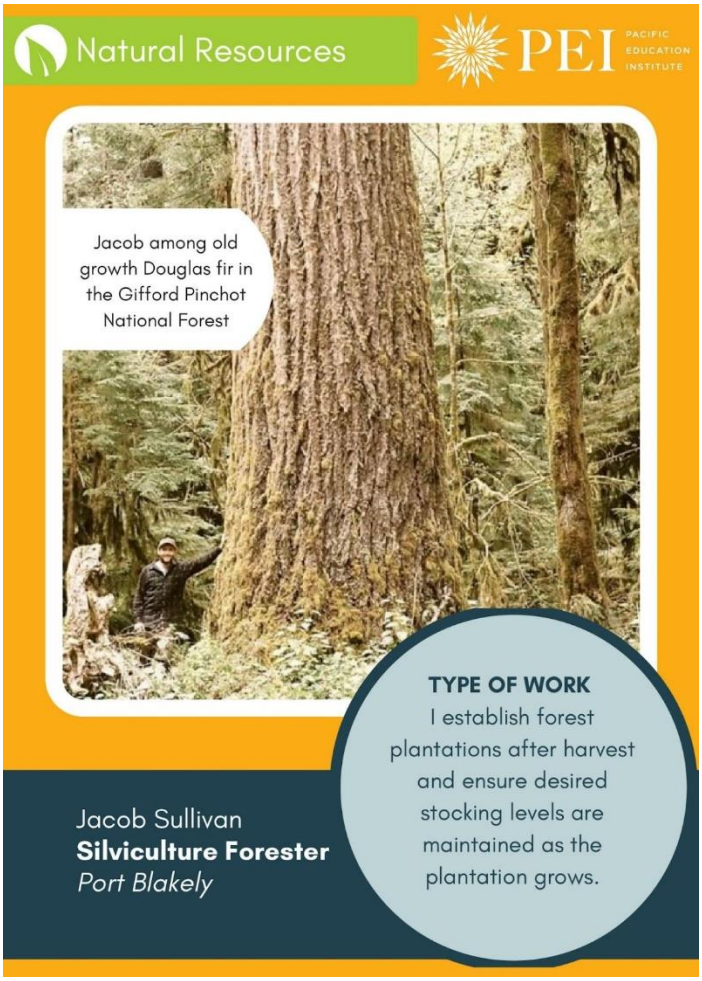
I can see connections between _____ and _____ because _____ .

So, what I think _____ is saying is that _____ . Is that correct?

Additional Resources

Career Connections

Jacob Sullivan's career profile card can also be found on the PEI website at https://pacificeducationinstitute.org/wp-content/uploads/2020/10/CPC-NR-JacobSullivan.pdf.



Community Resources Connection

Students may be unfamiliar with timber replantation. This collection of resources could help make the scenario more accessible:

- from Rayonier Inc. - Planting Millions of Trees Per-Year: How Rayonier Replants in the U.S. South and Pacific Northwest, 2021: youtu.be/GkBbfyfhd-w
- from WA Department of Natural Resources - Replanting Forest Seedlings, 2019: https://www.dnr.wa.gov/publications/frc_webster_plantingforestseedlings.pdf
- from Washington Forest Protection Association - Reforestation brings new life: https://www.wfpa.org/sustainable-forestry/reforestation/